```
Clear["Global`*"]
Remove["Global`*"]
CoefficientMatrix[listofequations_,
    paramlistleftside_, paramlistrightside_] := Module[{t},
    Print[Normal@CoefficientArrays[listofequations, paramlistleftside][[2]] //
      MatrixForm, ".", paramlistleftside // MatrixForm, "==",
     Normal@CoefficientArrays[listofequations, paramlistrightside][[2]] //
      MatrixForm, ".", paramlistrightside // MatrixForm]
  ];
θ1[t_] := a10 + a11 t + a12 t^2 + a13 t^3
θ2[t_] := a20 + a21 t + a22 t^2 + a23 t^3
ω1[t_] := Evaluate[D[θ1[t], t]]
acc1[t_] := Evaluate[D[θ1[t], {t, 2}]]
ω2[t_] := Evaluate[D[θ2[t], t]]
acc2[t_] := Evaluate[D[θ2[t], {t, 2}]]

θo = 0;
θv = 10 Degree;
θg = 15 Degree;
ω0 = 0;
ωg = 0;
t0 = 0;
tf1 := 2;
tf2 := 2;
equations = {θo == θ1[0], θv == θ1[tf1], θv == θ2[0], θg == θ2[tf2],
  ω0 == ω1[0], ωg = 0 == ω2[tf2], ω1[tf1] == ω2[0], acc1[tf1] == acc2[0]}
```

$$\{0 == a_{10},\ 10\,^\circ == a_{10} + 2\,a_{11} + 4\,a_{12} + 8\,a_{13},\ 10\,^\circ == a_{20},\ 15\,^\circ == a_{20} + 2\,a_{21} + 4\,a_{22} + 8\,a_{23},\ 0 == a_{11},\ 0 == a_{21} + 4\,a_{22} + 12\,a_{23},\ a_{11} + 4\,a_{12} + 12\,a_{13} == a_{21},\ 2\,a_{12} + 12\,a_{13} == 2\,a_{22}\}$$

```
Solve[equations, {a10, a11, a12, a13, a20, a21, a22, a23}] // N
```

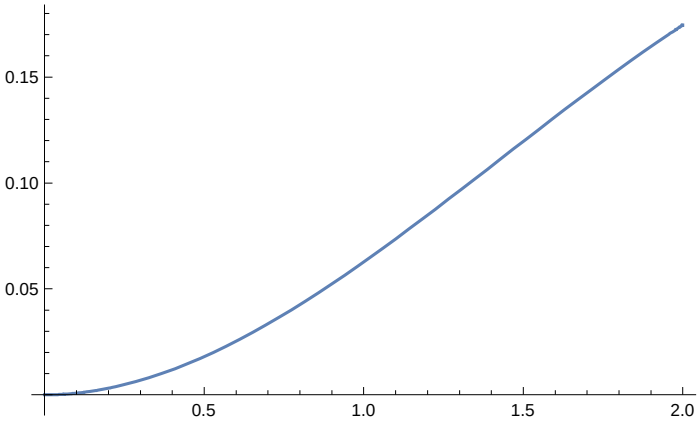
$$\{\{a_{10} \to 0.,\ a_{11} \to 0.,\ a_{12} \to 0.0818123,\ a_{13} \to -0.0190895,\ a_{20} \to 0.174533,\ a_{21} \to 0.0981748,\ a_{22} \to -0.0327249,\ a_{23} \to 0.00272708\}\}$$

```
{a10, a11, a12, a13, a20, a21, a22, a23} /. %198
```

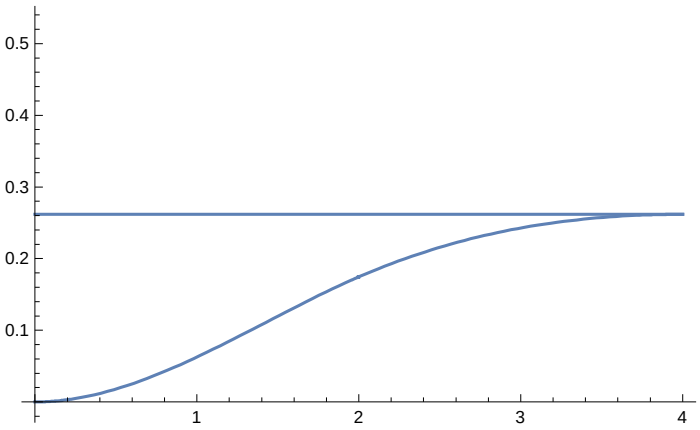
$$\{\{0., 0., 0.0818123, -0.0190895, 0.174533, 0.0981748, -0.0327249, 0.00272708\}\}$$

```
θ10[t_] := θ1[t] /.
  {a10 → 0.`, a11 → 0.`, a12 → 0.0818123086872342`, a13 → -0.019089538693687978`,
   a20 → 0.17453292519943295`, a21 → 0.09817477042468103`,
   a22 → -0.032724923474893676`, a23 → 0.00272707695624114`}
θ20[t_] := θ2[t] /. {a10 → 0.`, a11 → 0.`, a12 → 0.0818123086872342`,
   a13 → -0.019089538693687978`,
   a20 → 0.17453292519943295`, a21 → 0.09817477042468103`,
   a22 → -0.032724923474893676`, a23 → 0.00272707695624114`}
```

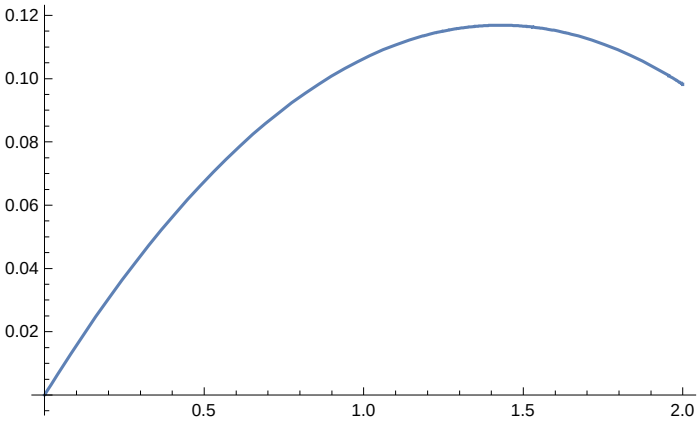
```
Plot[θ10[t], {t, 0, 2}]
```

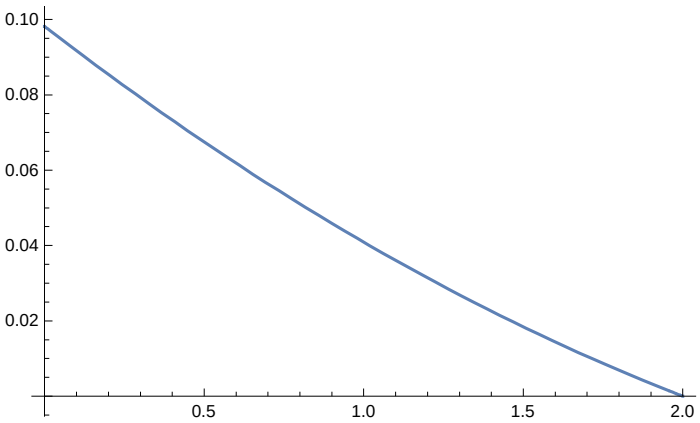



```
Show[Plot[15 Degree, {t, 0, 4}],
 Plot[θ10[t], {t, 0, 2}], Plot[θ20[t - 2], {t, 2, 4}]]
```

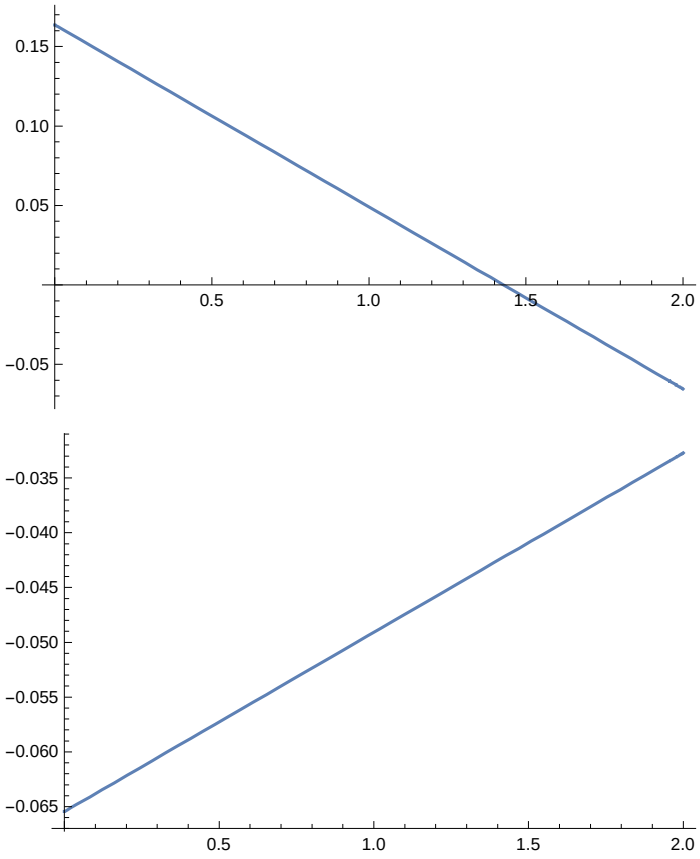


```
Plot[ω1[t] /.
  {a10 → 0.`, a11 → 0.`, a12 → 0.0818123086872342`, a13 → -0.019089538693687978`,
   a20 → 0.17453292519943295`, a21 → 0.09817477042468103`,
   a22 → -0.032724923474893676`, a23 → 0.00272707695624114`}, {t, 0, 2}]
Plot[ω2[t] /. {a10 → 0.`, a11 → 0.`, a12 → 0.0818123086872342`,
   a13 → -0.019089538693687978`, a20 → 0.17453292519943295`,
   a21 → 0.09817477042468103`, a22 → -0.032724923474893676`,
   a23 → 0.00272707695624114`}, {t, 0, 2}]
```





```
Plot[acc1[t] /.
  {a10 → 0.`, a11 → 0.`, a12 → 0.0818123086872342`, a13 → -0.019089538693687978`,
   a20 → 0.17453292519943295`, a21 → 0.09817477042468103`,
   a22 → -0.032724923474893676`, a23 → 0.00272707695624114`}, {t, 0, 2}]
Plot[acc2[t] /. {a10 → 0.`, a11 → 0.`, a12 → 0.0818123086872342`,
   a13 → -0.019089538693687978`, a20 → 0.17453292519943295`,
   a21 → 0.09817477042468103`, a22 → -0.032724923474893676`,
   a23 → 0.00272707695624114`}, {t, 0, 2}]
```




```
Px == r1 Cos[ϕ1] + r2 Cos[ϕ2]
Py == r1 Sin[ϕ1] + r2 Sin[ϕ2]
```

```
Px == r1 Cos[ϕ1] + r2 Cos[ϕ2]
Py == r1 Sin[ϕ1] + r2 Sin[ϕ2]
```

```
Eliminate[{Px == r1 Cos[ϕ1] + r2 Cos[ϕ2], Py == r1 Sin[ϕ1] + r2 Sin[ϕ2]}, {ϕ2}] //
 Simplify
```

Eliminate::ifun : Inverse functions are being used by Eliminate,

so some solutions may not be found; use Reduce for complete solution information. >>

$$Px^2 + Py^2 + r1^2 == r2^2 + 2\,Px\,r1\,Cos[\phi1] + 2\,Py\,r1\,Sin[\phi1]$$

```
Solve[Px^2 + Py^2 + r1^2 == r2^2 + 2 Px r1 Cos[ϕ1] + 2 Py r1 Sin[ϕ1], ϕ1] // Simplify
```

```
{{ϕ1 →
    ConditionalExpression[ArcTan[((Px^3 r1 + Px Py^2 r1 + Px r1^3 - Px r1 r2^2 - Sqrt[-Py^2 r1^2
              (Px^4 + Py^4 + 2 Px^2 (Py^2 - r1^2 - r2^2) + (r1^2 - r2^2)^2 - 2 Py^2 (r1^2 + r2^2))]))/
        ((Px^2 + Py^2) r1^2), ((Px^2 Py^2 r1 + Py^4 r1 + Py^2 r1^3 - Py^2 r1 r2^2 + Px Sqrt[-Py^2
              r1^2 (Px^4 + Py^4 + 2 Px^2 (Py^2 - r1^2 - r2^2) + (r1^2 - r2^2)^2 - 2 Py^2 (r1^2 + r2^2))]))/
        (Py (Px^2 + Py^2) r1^2)] + 2 π C[1], C[1] ∈ Integers],
  {ϕ1 → ConditionalExpression[ArcTan[((Px^3 r1 + Px Py^2 r1 + Px r1^3 - Px r1 r2^2 + Sqrt[-Py^2
              r1^2 (Px^4 + Py^4 + 2 Px^2 (Py^2 - r1^2 - r2^2) + (r1^2 - r2^2)^2 - 2 Py^2 (r1^2 + r2^2))]))/
        ((Px^2 + Py^2) r1^2), ((Px^2 Py^2 r1 + Py^4 r1 + Py^2 r1^3 - Py^2 r1 r2^2 - Px Sqrt[-Py^2
              r1^2 (Px^4 + Py^4 + 2 Px^2 (Py^2 - r1^2 - r2^2) + (r1^2 - r2^2)^2 - 2 Py^2 (r1^2 + r2^2))]))/
        (Py (Px^2 + Py^2) r1^2)] + 2 π C[1], C[1] ∈ Integers]}}
```

```
Clear["Global`*"]
Remove["Global`*"]
r1 = 10;
r2 = 8;
Px = 0;
Py = 3;

Solve[
 {Px == r1 Cos[ϕ1] + r2 Cos[ϕ1 + ψ1], Py == r1 Sin[ϕ1] + r2 Sin[ϕ1 + ψ1]} // N, {ϕ1, ψ1}]
```

Solve::ifun : Inverse functions are being used by Solve, so some solutions may not be found; use Reduce for complete solution information. >>

```
{{ϕ1 → 0.848062, ψ1 → 2.89094}, {ϕ1 → 2.29353, ψ1 → -2.89094}}
```

```
Pxlist = {3, 5, 6, 7, 8, 9, 10, 12, 11, 8};
Pylist = {1, 1, 1, 1, 1, 1, 1, 1, 1, 1};
Do[Px = Pxlist[[i]];
 Py = Pylist[[i]];
 sol =
  Solve[{Px == r1 Cos[ϕ1] + r2 Cos[ϕ1 + ψ1], Py == r1 Sin[ϕ1] + r2 Sin[ϕ1 + ψ1]} // N,
    {ϕ1, ψ1}];
 Print[sol[[1]]]
 , {i, 1, 10}]
```

Solve::ifun : Inverse functions are being used by Solve, so some solutions may not be found; use Reduce for complete solution information. >>

$\{\phi 1 \to -0.434631, \psi 1 \to 2.86687\}$

Solve::ifun : Inverse functions are being used by Solve, so some solutions may not be found; use Reduce for complete solution information. >>

$\{\phi 1 \to -0.719912, \psi 1 \to 2.61099\}$

Solve::ifun : Inverse functions are being used by Solve, so some solutions may not be found; use Reduce for complete solution information. >>

General::stop : Further output of Solve::ifun will be suppressed during this calculation. >>

$\{\phi 1 \to -0.762076, \psi 1 \to 2.48775\}$

$\{\phi 1 \to -0.775219, \psi 1 \to 2.36385\}$

$\{\phi 1 \to -0.769547, \psi 1 \to 2.23795\}$

$\{\phi 1 \to -0.750519, \psi 1 \to 2.10889\}$

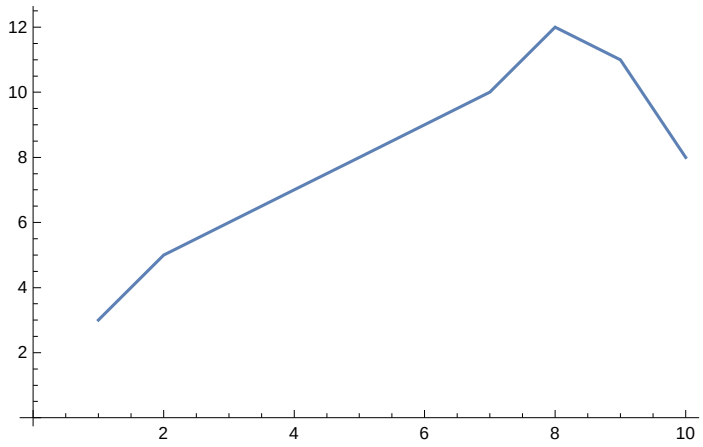
$\{\phi 1 \to -0.72118, \psi 1 \to 1.9755\}$

$\{\phi 1 \to -0.637229, \psi 1 \to 1.68983\}$

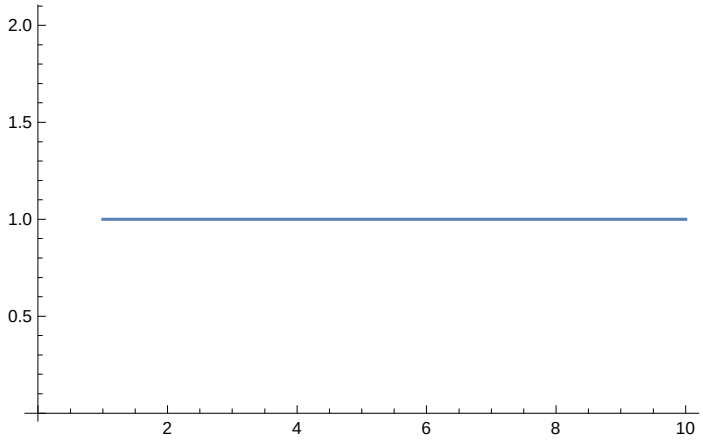
$\{\phi 1 \to -0.68318, \psi 1 \to 1.83641\}$

$\{\phi 1 \to -0.769547, \psi 1 \to 2.23795\}$

```
ListLinePlot[Pxlist]
```



```
ListLinePlot[Pylist]
```




```
0 == r1 Cos[ϕ1[t]] + r2 Cos[ϕ1[t] + ψ1[t]] - Px[t]
0 == r1 Sin[ϕ1[t]] + r2 Sin[ϕ1[t] + ψ1[t]] - Py[t]
```

```
D[0 == r1 Cos[ϕ1[t]] + r2 Cos[ϕ1[t] + ψ1[t]] - Px[t], t]
D[0 == r1 Sin[ϕ1[t]] + r2 Sin[ϕ1[t] + ψ1[t]] - Py[t], t]
```

```
0 == -Px'[t] - r1 Sin[ϕ1[t]] ϕ1'[t] - r2 Sin[ϕ1[t] + ψ1[t]] (ϕ1'[t] + ψ1'[t])
```

```
0 == -Py'[t] + r1 Cos[ϕ1[t]] ϕ1'[t] + r2 Cos[ϕ1[t] + ψ1[t]] (ϕ1'[t] + ψ1'[t])
```

```
Solve[{0 == -Px'[t] - r1 Sin[ϕ1[t]] ϕ1'[t] - r2 Sin[ϕ1[t] + ψ1[t]] (ϕ1'[t] + ψ1'[t]),
    0 == -Py'[t] + r1 Cos[ϕ1[t]] ϕ1'[t] + r2 Cos[ϕ1[t] + ψ1[t]] (ϕ1'[t] + ψ1'[t])},
  {ϕ1'[t], ψ1'[t]}] /. t → i
```

```
{{ϕ1'[i] → -((Cos[ϕ1[i] + ψ1[i]] Px'[i] + Sin[ϕ1[i] + ψ1[i]] Py'[i]) /
       (r1 (Cos[ϕ1[i] + ψ1[i]] Sin[ϕ1[i]] - Cos[ϕ1[i]] Sin[ϕ1[i] + ψ1[i]]))),
  ψ1'[i] → -((-r2 Cot[ϕ1[i] + ψ1[i]] Px'[i] - r1 Cos[ϕ1[i]] Csc[ϕ1[i] + ψ1[i]] Px'[i] -
         r2 Py'[i] - r1 Csc[ϕ1[i] + ψ1[i]] Sin[ϕ1[i]] Py'[i]) /
       (r1 r2 (-Cos[ϕ1[i]] + Cot[ϕ1[i] + ψ1[i]] Sin[ϕ1[i]])))}}
```

```
Clear["Global`*"]
Remove["Global`*"]
r1 = 10;
r2 = 8;
Pxlist = {3, 5, 6, 7, 8, 9, 10, 12, 11, 8};
Pylist = {1, 1, 5, 1, 6, 1, 1, 1, 1, 1};
Do[Vx[i] = 4, {i, 1, 10}];
Do[Vy[i] = 5, {i, 1, 10}];
Do[Px = Pxlist[[i]];
 Py = Pylist[[i]];
 sol =
  Solve[{Px == r1 Cos[ϕ1] + r2 Cos[ϕ1 + ψ1], Py == r1 Sin[ϕ1] + r2 Sin[ϕ1 + ψ1]} // N,
   {ϕ1, ψ1}];
 ϕ1[i] = sol[[1, 1, 2]];
 ψ1[i] = sol[[2, 1, 2]];
 Print[{{ϕ1'[i] → -((Cos[ϕ1[i] + ψ1[i]] Vx[i] + Sin[ϕ1[i] + ψ1[i]] Vy[i]) /
         (r1 (Cos[ϕ1[i] + ψ1[i]] Sin[ϕ1[i]] - Cos[ϕ1[i]] Sin[ϕ1[i] + ψ1[i]]))),
    ψ1'[i] → -((-r2 Cot[ϕ1[i] + ψ1[i]] Vx[i] - r1 Cos[ϕ1[i]] Csc[ϕ1[i] + ψ1[i]]
           Vx[i] - r2 Vy[i] - r1 Csc[ϕ1[i] + ψ1[i]] Sin[ϕ1[i]] Vy[i]) /
         (r1 r2 (-Cos[ϕ1[i]] + Cot[ϕ1[i] + ψ1[i]] Sin[ϕ1[i]])))}}];
 ,
 {i,
  1,
  10}]
```

Solve::ifun : Inverse functions are being used by Solve, so some solutions may not be found; use Reduce for complete solution information. >>

{{ϕ1'[1] → 0.703685, ψ1'[1] → -0.919716}}

Solve::ifun : Inverse functions are being used by Solve, so some solutions may not be found; use Reduce for complete solution information. >>

{{ϕ1'[2] → 0.625474, ψ1'[2] → -0.585217}}

Solve::ifun : Inverse functions are being used by Solve, so some solutions may not be found; use Reduce for complete solution information. >>

General::stop : Further output of Solve::ifun will be suppressed during this calculation. >>

{{ϕ1'[3] → 0.564107, ψ1'[3] → -0.925757}}

{{ϕ1'[4] → 0.601005, ψ1'[4] → -0.50892}}

{{ϕ1'[5] → 0.595232, ψ1'[5] → -0.977667}}

{{ϕ1'[6] → 0.605385, ψ1'[6] → -0.53202}}

{{ϕ1'[7] → 0.617012, ψ1'[7] → -0.570329}}

{{ϕ1'[8] → 0.663021, ψ1'[8] → -0.704723}}

{{ϕ1'[9] → 0.63568, ψ1'[9] → -0.626802}}

{{ϕ1'[10] → 0.600062, ψ1'[10] → -0.511048}}

```
Px'[2]
```

0

```
P
```

Do::itform : Argument 1 at position 3 does not have the correct form for an iterator. >>

Do::itform : Argument 1 at position 3 does not have the correct form for an iterator. >>

Solve::ifun : Inverse functions are being used by Solve, so
some solutions may not be found; use Reduce for complete solution information. >>

$$\{\{\phi 1 \to -0.434631,\ \psi 1 \to 2.86687\},\ \{\phi 1 \to 1.07813,\ \psi 1 \to -2.86687\}\}$$

$$\{\{\phi 1'[1] \to 0.,\ \psi 1'[1] \to 0.\}\}$$

Solve::ifun : Inverse functions are being used by Solve, so
some solutions may not be found; use Reduce for complete solution information. >>

$$\{\{\phi 1 \to -0.719912,\ \psi 1 \to 2.61099\},\ \{\phi 1 \to 1.1147,\ \psi 1 \to -2.61099\}\}$$

$$\{\{\phi 1'[2] \to 0.,\ \psi 1'[2] \to 0.\}\}$$

Solve::ifun : Inverse functions are being used by Solve, so
some solutions may not be found; use Reduce for complete solution information. >>

General::stop : Further output of Solve::ifun will be suppressed during this calculation. >>

$$\{\{\phi 1 \to -0.206067,\ \psi 1 \to 2.27019\},\ \{\phi 1 \to 1.59554,\ \psi 1 \to -2.27019\}\}$$

$$\{\{\phi 1'[3] \to 0.,\ \psi 1'[3] \to 0.\}\}$$

$$\{\{\phi 1 \to -0.775219,\ \psi 1 \to 2.36385\},\ \{\phi 1 \to 1.05901,\ \psi 1 \to -2.36385\}\}$$

$$\{\{\phi 1'[4] \to 0.,\ \psi 1'[4] \to 0.\}\}$$

$$\{\{\phi 1 \to -0.179533,\ \psi 1 \to 1.98231\},\ \{\phi 1 \to 1.46653,\ \psi 1 \to -1.98231\}\}$$

$$\{\{\phi 1'[5] \to 0.,\ \psi 1'[5] \to 0.\}\}$$

$$\{\{\phi 1 \to -0.750519,\ \psi 1 \to 2.10889\},\ \{\phi 1 \to 0.971833,\ \psi 1 \to -2.10889\}\}$$

$$\{\{\phi 1'[6] \to 0.,\ \psi 1'[6] \to 0.\}\}$$

$$\{\{\phi 1 \to -0.72118,\ \psi 1 \to 1.9755\},\ \{\phi 1 \to 0.920517,\ \psi 1 \to -1.9755\}\}$$

$$\{\{\phi 1'[7] \to 0.,\ \psi 1'[7] \to 0.\}\}$$

$$\{\{\phi 1 \to -0.637229,\ \psi 1 \to 1.68983\},\ \{\phi 1 \to 0.803511,\ \psi 1 \to -1.68983\}\}$$

$$\{\{\phi 1'[8] \to 0.,\ \psi 1'[8] \to 0.\}\}$$

$$\{\{\phi 1 \to -0.68318,\ \psi 1 \to 1.83641\},\ \{\phi 1 \to 0.8645,\ \psi 1 \to -1.83641\}\}$$

$$\{\{\phi 1'[9] \to 0.,\ \psi 1'[9] \to 0.\}\}$$

$$\{\{\phi 1 \to -0.769547,\ \psi 1 \to 2.23795\},\ \{\phi 1 \to 1.01826,\ \psi 1 \to -2.23795\}\}$$

$$\{\{\phi 1'[10] \to 0.,\ \psi 1'[10] \to 0.\}\}$$

```
{{ϕ1'[i] → -((Cos[ϕ1[i] + ψ1[i]] Px'[i] + Sin[ϕ1[i] + ψ1[i]] Py'[i]) /
        (r1 (Cos[ϕ1[i] + ψ1[i]] Sin[ϕ1[i]] - Cos[ϕ1[i]] Sin[ϕ1[i] + ψ1[i]]))),
   ψ1'[i] → -((-r2 Cot[ϕ1[i] + ψ1[i]] Px'[i] - r1 Cos[ϕ1[i]] Csc[ϕ1[i] + ψ1[i]]
          Px'[i] - r2 Py'[i] - r1 Csc[ϕ1[i] + ψ1[i]] Sin[ϕ1[i]] Py'[i]) /
        (r1 r2 (-Cos[ϕ1[i]] + Cot[ϕ1[i] + ψ1[i]] Sin[ϕ1[i]])))}} /. i → 1
```

$$\{\{\phi 1'[1] \to 0,\ \psi 1'[1] \to 0\}\}$$